INVERSE PROBLEMS

MATHEMATICAL AND ANALYTICAL TECHNIQUES WITH APPLICATIONS TO ENGINEERING

Alexander G. Ramm

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To Luba and Olga

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FOREWORD

The importance of mathematics in the study of problems arising from the real world, and the increasing success with which it has been used to model situations ranging from the purely deterministic to the stochastic, is well established. The purpose of the set of volumes to which the present one belongs is to make available authoritative, up to date, and self-contained accounts of some of the most important and useful of these analytical approaches and techniques. Each volume provides a detailed introduction to a specific subject area of current importance that is summarized below, and then goes beyond this by reviewing recent contributions, and so serving as a valuable reference source.

The progress in applicable mathematics has been brought about by the extension and development of many important analytical approaches and techniques, in areas both old and new, frequently aided by the use of computers without which the solution of realistic problems would otherwise have been impossible.

A case in point is the analytical technique of singular perturbation theory which has a long history. In recent years it has been used in many different ways, and its importance has been enhanced by it having been used in various fields to derive sequences of asymptotic approximations, each with a higher order of accuracy than its predecessor. These approximations have, in turn, provided a better understanding of the subject and stimulated the development of new methods for the numerical solution of the higher order approximations. A typical example of this type is to be found in the general study of nonlinear wave propagation phenomena as typified by the study of water waves.

Elsewhere, as with the identification and emergence of the study of inverse problems, new analytical approaches have stimulated the development of numerical techniques for the solution of this major class of practical problems. Such work divides naturally into two parts, the first being the identification and formulation of inverse problems, the theory of ill-posed problems and the class of one-dimensional inverse problems, and the second being the study and theory of multidimensional inverse problems.

On occasions the development of analytical results and their implementation by computer have proceeded in parallel, as with the development of the fast boundary element methods necessary for the numerical solution of partial differential equations in several dimensions. This work has been stimulated by the study of boundary integral equations, which in turn has involved the study of boundary elements, collocation methods, Galerkin methods, iterative methods and others, and then on to their implementation in the case of the Helmholtz equation, the Lamé equations, the Stokes equations, and various other equations of physical significance.

A major development in the theory of partial differential equations has been the use of group theoretic methods when seeking solutions, and in the introduction of the comparatively new method of differential constraints. In addition to the useful contributions made by such studies to the understanding of the properties of solutions, and to the identification and construction of new analytical solutions for well established equations, the approach has also been of value when seeking numerical solutions. This is mainly because of the way in many special cases, as with similarity solutions, a group theoretic approach can enable the number of dimensions occurring in a physical problem to be reduced, thereby resulting in a significant simplification when seeking a numerical solution in several dimensions. Special analytical solutions found in this way are also of value when testing the accuracy and efficiency of new numerical schemes.

A different area in which significant analytical advances have been achieved is in the field of stochastic differential equations. These equations are finding an increasing number of applications in physical problems involving random phenomena, and others that are only now beginning to emerge, as is happening with the current use of stochastic models in the financial world. The methods used in the study of stochastic differential equations differ somewhat from those employed in the applications mentioned so far, since they depend for their success on the Ito calculus, martingale theory and the Doob-Meyer decomposition theorem, the details of which are developed as necessary in the volume on stochastic differential equations.

There are, of course, other topics in addition to those mentioned above that are of considerable practical importance, and which have experienced significant developments in recent years, but accounts of these must wait until later.

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PREFACE

This book can be used for courses at various levels in ill-posed problems and inverse problems. The bibliography of the subject is enormous. It is not possible to compile a complete bibliography and no attempt was made to do this. The bibliography contains some books where the reader will find additional references. The author has used extensively his earlier published papers, and referenced these, as well as the papers of other authors that were used or mentioned.

Let us outline some of the novel features in this book.

In Chapter 1 the statement of various inverse problems is given.

In Chapter 2 the presentation of the theory of ill-posed problems is shorter and sometimes simpler than that published earlier, and quite a few new results are included. Regularization for ill-posed operator equations with unbounded nonlinear operators is studied. A novel version of the discrepancy principle is formulated for nonlinear operator equations. Convergence rate estimates are given for Backus-Gilbert-type methods. The DSM (Dynamical systems method) in ill-posed problems is presented in detail. The presentation is based on the author's papers and the joint papers of the author and his students. These results appear for the first time in book form. Papers [R216], [R217], [R218], [R220], [ARS3], [AR1] have been used in this Chapter.

In Chapter 3 the presentation of one-dimensional inverse problems is based mostly on the author's papers, especially on [R221]. It contains many novel results, which are described at the beginning of the Chapter. The presentation of the classical results, for example, Gel'fand-Levitan's theory, and Marchenko's theory, contains many novel points. The presentation of M. G. Krein's inversion theory with complete proofs is given for the first time. The Newton-Sabatier inversion theory, which has been in the literature for more than 40 years, and was presented in two monographs [CS], [N], is analyzed and shown to be fundamentally wrong in the sense that its foundations are wrong (cf. [R206]). This Chapter is based on the papers [R221], [R199], [R197], [R196], [R195], [R192], [R185]. One of the first papers on inverse spectral problems was Ambartsumian's paper (1929) [Am], where it was proved that one spectrum determines the one-dimensional Neumann Schrödinger's operator uniquely. This result is an exceptional one: in general one spectrum does not determine the potential uniquely (see Section 3.7 and [PT]). Only 63 years later a multidimensional analog of Ambartsumian's result was obtained ([RSt1]). The main technical tool in this Chapter and in Chapter 5 is Property C, that is, completeness of the set of products of solutions to homogeneous differential equations. For partial differential equations this tool has been introduced in [R87] and developed in many papers and in the monograph [R139]. For ordinary differential equations completeness of the products of solutions to homegeneous ordinary equations has been used in different forms in [B], [L1]. In our book Property C for ODE is presented in the form introduced and developed by the author in [R196].

In Chapter 4 the presentation of inverse obstacle scattering problems contains many novel points. The requirements on the smoothness of the boundary are minimal, stability estimates for the inversion procedure corresponding to fixed-frequency data are given, the high-frequency inversion formulas are discussed and the error of the inversion from noisy data is estimated. Analysis of the currently used numerical methods is given. This Chapter is based on [R83], [R155], [R162], [R164], [R167], [R167], [R171], [RSa].

In Chapter 5 a presentation of the solution of the 3D inverse potential scattering problem with fixed-energy noisy data is given. This Chapter is based on the series of the author's papers, especially on the paper [R203]. The basic concept used in the analysis of the inverse scattering problem in Chapter 5 is the concept of Property C, i.e., completeness of the set of products of solutions to homogeneous partial differential equations. This concept was introduced by the author ([R87]) and applied to many inverse problems (see [R139] and references therein). An important part of the theory consists of obtaining stability estimates for the potential, reconstructed from fixed-energy noisy data (and from exact data). Error estimates for the Born inversion are given under suitable assumptions. It is shown that the Born inversion may fail while the Born approximation works well. In other words, the Born approximation may be applicable for solving the direct scattering problem, while the Born inversion, that is, inversion based on the Born approximation, may fail. The Born inversion is still popular in applications, therefore these error estimates will hopefully be useful for practitioners.

The author's inversion method for fixed-energy scattering data, is compared with that based on the usage of the Dirichlet-to-Neumann map. The author shows why the difficulties in numerical implementation of his method are less formidable than the difficulties in implementing the inversion method based on the Dirichlet-to-Neumann map.

A necessary and sufficient condition for a scatterer to be spherically symmetric is given ([R128]).

In Chapter 6 an example of non-uniqueness of the solution to a 3D problem of geophysics is given. It illustrates the crucial role of the uniqueness theorems in a study of inverse problems. One may try to solve numerically such a problem, by a parameter-fitting, which is very popular among practitioners. But if the uniqueness result is not established, the numerical results may be meaningless. Some uniqueness theorems for inverse boundary value problem and for an inverse problem for hyperbolic equations are established in this Chapter.

In Chapter 7, inverse problems of potential theory and antenna synthesis are briefly discussed. The presentation of the theory on this topic is not complete: there are books and many papers on antenna synthesis (e.g., [MJ], [ZK], [AVST], [R21], [R26]) including nonlinear problems of antenna synthesis [R23], [R27]).

Chapter 8 contains a discussion of non-overdetermined problems. These are, roughly speaking, the inverse problems in which the unknown function depends on the same number of variables as the data function. Examples of such problems are given. Most of these problems are open: even uniqueness theorems are not available. Such a problem, namely, recovery of an unknown coefficient in a Schrödinger equation in a bounded domain from the knowledge of the values of the spectral function $\rho(s, s, \lambda)$ on the boundary is discussed under the assumption that all the eigenvalues are simple, that is, the corresponding eigenspaces are one-dimensional. The presentation follows [R198].

In Chapter 9 the theory of the inversion of low-frequency data is presented. This theory is based on the series of author's papers, starting with [R68], [R77], and uses the presentation in [R83] and [R139]. Almost all of the results in this Chapter are from the above papers and books.

Chapter 10 is a summary of the author's results regarding the theory of wave scattering by small bodies of arbitrary shapes. These results have been obtained in a series of the author's papers and are summarized in [R65], [R50]. The solution of inverse radiomeasurements problem ([R33], [R65]) is based on these results. Also, these results are used in the solution of the problem of finding small subsurface inhomogeneities from the scattering data, measured on the surface. The solution to this problem can be used in modeling ultrasound mammography, in finding small holes in metallic objects, and in many other applied problems.

In Chapter 11 the classical Pompeiu problem is presented following the papers [R177], [R186].

The author thanks several publishers of his papers, mentioned above, for the permission to use these papers in the book.

There are many questions that the author did not discuss in this book: inverse scattering for periodic potentials and other periodic objects, such as gratings, periodic objects, (see, e.g., [L] for one-dimensional scattering problems for periodic potentials), the Carleman estimates and their applications to inverse problems ([Bu2], [H], [LRS]), the inverse problems for elasticity and Maxwell's equations ([RK], [Ya]), the methods based on controllability results ([Bel]), problems of tomography and integral geometry ([RKa], [R139]), etc. Numerous parameter-fitting schemes for solving various